Recursive Sequences

Friday, April 7, 2023 9:36 AM

initial condition: this choice affects convergence values / will end up going towards same limit

problem: $a_n = \sqrt[3]{a_{n-1} + 6}$, $a_1 = 1 \longrightarrow$ convergent? if so, limit?

solution: 1) get feeling: a_1 a_2 a_3 a_4 a_5 a_6 ...

1 $\quad \sqrt[3]{a_1 + 6}$ $\quad \sqrt[3]{a_2 + 6}$ ...

$\sqrt[3]{7}$ $\quad \sqrt[3]{\sqrt[3]{7} + 6}$

$\approx 1.9\ldots$ $\quad \approx 1.999\ldots$

seems to increase

let's show ...

1) a_n is bounded above: show $a_n < 2$

$1 = a_1 \leq 2$ ✓, $\quad a_n = \sqrt[3]{\underbrace{\underbrace{a_{n-1}}_{<2} + 6}_{<8}} < \sqrt[3]{8} < 2$ ✓ $\nearrow$ MCT = converges

2) a_n increasing: we want $a_{n-1} \leq a_n = \sqrt[3]{a_{n-1} + 6}$

a_{n-1} greater each time

$= \sqrt[3]{x}$ (increases)

$\downarrow$

means 2 is limit

true bc $+6 \uparrow$ & $\sqrt[3]{6} \uparrow$

how to compute L of recursive a_n?

- plug L where it says a_n for any n

problem: $\left.\begin{matrix} a_n \to L \\ a_{n-1} \to L \end{matrix}\right\}$ $a_n = \sqrt[3]{a_{n-1} + 6}$ means $L = \sqrt[3]{L + 6} = \boxed{2}$